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INTERFEROMETRIC STUDY OF THE TEMPERATURE FIELD IN THE NATURAL CONVECTIVE FLOW ALONG A HEATED VERTICAL CYLINDER

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NOMENCLATURE

- Gr, Grashof number;
- r, radial coordinate;
- r_o, radius of cylinder;
- T, absolute temperature of air in the convective flow;
- T_w, constant wall temperature;
- T_∞, temperature of ambient air;
- z, vertical or axial coordinate;
- η, non-dimensional radial coordinate;
- Θ, $= (T-T_\infty)/(T_w-T_\infty)$;
- ξ, non-dimensional axial coordinate.

INTRODUCTION

OPTICAL interferometry is a well established method for the quantitative study of the density field in natural convective flows [1]. With these techniques, no probe is inserted into the flow which is very sensitive to external disturbances. Sernas and Fletcher [2] applied a simple Wollaston prism schlieren interferometer [3, 4] to the study of the natural convective flow along a heated, vertical flat plate. This interferometric system can be classified as a shearing interferometer [5] and responds to changes of the gas density gradient in the object field. For the case of a two-dimensional test object, like the flow along the heated flat plate, the observed shift of the interference fringes is directly proportional to the density gradient normal to the undisturbed fringes, or proportional to the temperature gradient in the case of the natural convective flow. Hence, this is an easy way to determine the heat-transfer coefficient at the heated vertical wall.

In the case of a test field with an axisymmetric density distribution, the gas density varies along the path of a light ray in the interferometric system, and the information on the flow field is integrated along the path of the light ray. The aim of the present investigations was to study the temperature field in the natural convective air flow along a heated, vertical cylinder, which represents such an axisymmetric optical test field, and to compare the measured temperature profiles with existing theoretical predictions.

MEASUREMENTS AND COMPARISON WITH THEORY

The cylinders investigated were made of copper and filled with water which was held at a constant temperature about 40°C above the ambient room temperature. The convective air flow along the outer wall of the cylinder can then be regarded to occur along a constant temperature wall. This assumption is supported by Sernas' and Fletcher's measurements performed with thermocouples [2]. Several cylinders were tested, all having the same height of 25 cm. The Wollaston prism shearing interferometer used for the measurements was operated with parallel light through the test field, the light direction being normal to the cylinder axis. Figure 1 is a typical interferogram which shows the distortion of the interference fringes in the zone of varying gas density close to the cylinder wall. A numerical procedure for the evaluation of such an axisymmetric shearing interferogram is described in [6]. This procedure yields the density as a function of the axisymmetric coordinate r, and since the pressure is assumed here to be constant in a horizontal plane z = constant, one easily obtains the temperature distribution $T = T(r)$ in a cross-section z = constant normal to the cylinder axis.

In Fig. 2 the experimental results for three cylinders are plotted in a coordinate system which is chosen according to the coordinates used in the theoretical predictions of the temperature field. $\Theta = (T-T_\infty)/(T_w-T_\infty)$ is a non-dimensional temperature difference, where T designates the temperature at the radius r in a plane z = constant of the test field. The abscissa

$$\eta = \frac{1}{2^{3/2}} \cdot \frac{1}{z} \cdot \frac{r^2 - r_0^2}{r^2} (Gr_z)^{1/4}$$

is primarily the non-dimensional radial coordinate which also includes the axial coordinate z and the Grashof number Gr_z formed with the axial coordinate z. Θ decreases from the value $\Theta_w = 1$ at the cylinder wall to zero in a great distance from the cylinder. The measurements have been performed at various heights z for each cylinder.

The experimental results have been compared with theoretical predictions obtained by Sparrow and Gregg [7] and by Viskanta [8]. The temperature difference Θ is expressed as a power series of the non-dimensional axial coordinate

$$\Theta(\xi, \eta) = \Theta_0(\eta) + \xi \cdot \Theta_1(\eta) + \xi^2 \cdot \Theta_2(\eta) + \dots$$

with

$$\xi = \frac{2^{3/2}}{(Gr_z)^{1/4}} \cdot \frac{z}{r_0}.$$

The coefficients Θ_i depend on the coordinate η only. Θ_0 is the solution for the temperature field near the heated vertical flat plate. Only the zeroth order and the linear term have been evaluated in the representation of Fig. 2. The curves for cylinders with different radii do not coincide. The temperature curve for the cylinder with the greatest radius investigated ($r_0 = 10$ cm) nearly coincides with the solution for the flat plate. The relative error between the measured and calculated values is small. But there appears a general difference in the pattern of the experimental and theoretical curves. The experimental curves obviously experience a change in curvature near the cylinder wall which is not predicted by the theory. This discrepancy also exists for the flat plate case. Then, the maximum of the distorted

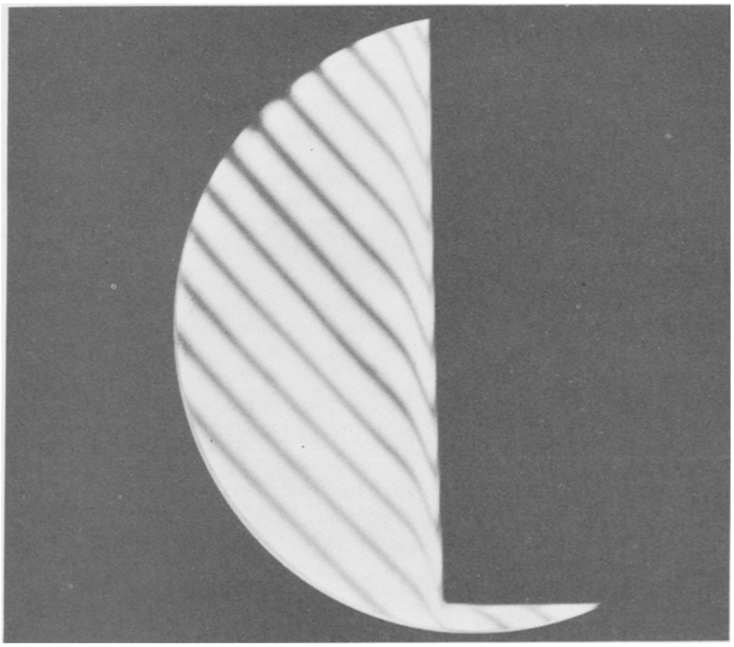

FIG. 1. Shearing interferogram obtained from the convective air flow along a heated vertical cylinder.

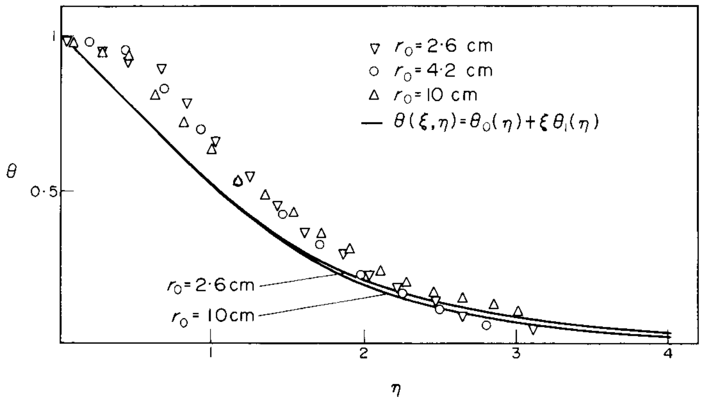

FIG. 2. Non-dimensional representation of the temperature field.

interference fringes near the wall (see Fig. 3 of [2]) exactly designates the position of the change of curvature in the temperature profiles. The temperature gradient at the wall, as obtained from the optical measurements, is therefore smaller than predicted by theory, and the same applies to the heat-transfer coefficient at the wall. A temperature profile with such a change in curvature is only predicted theoretically for a wall with a step discontinuity in surface temperature [9], but the thoroughful measurements of Sernas and Fletcher [2] exclude such an inhomogeneous wall temperature for the experiments reported in this note.

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ZERO LATENT ROOTS OF THE COEFFICIENT MATRIX IN THE EQUATION OF MULTICHANNEL EXCHANGERS

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NOMENCLATURE

a_{ij}, number of transfer units per unit of coordinate x;
e_i, column vector of $\mathbf{K}$;
h_{ij}, common perimeter of channels i and j;
k_{ij}, overall surface conductance for heat transfer between channels i and j;
n, number of channels;
t_i, temperature of fluid in channel i;
x, space coordinate along the channel;
$\boldsymbol{x}$, column vector;
$\mathbf{A}$, coefficient matrix;
$\mathbf{B}$, diagonal matrix;
$\mathbf{C}$, $n \times n$ matrix;
$\mathbf{S}$, $n \times n$ symmetric matrix;
$\boldsymbol{t}$, temperature column vector, $\boldsymbol{t} = [t_1, t_2, \ldots, t_n]^T$;
W, fluid heat capacity rate.

CONSIDER the equation

$$\frac{\mathrm{d}\boldsymbol{t}}{\mathrm{d}x} = \mathbf{A}\boldsymbol{t} \tag{1}$$

where $\boldsymbol{t}$ is the column matrix of temperature and $\mathbf{A}$ is a square, non-block diagonal matrix of order n and of rank equal to $n-1$ [1] defined by

$$\mathbf{A} = \begin{bmatrix} a_{11}a_{12}\ldots a_{1n} \\ \ldots\ldots\ldots\ldots \\ a_{n1}a_{n2}\ldots a_{nn} \end{bmatrix} \tag{2}$$

$$a_{ij} = \frac{k_{ij}h_{ij}}{W_i} \qquad (i \neq j,\ k_{ij}h_{ij} = k_{ji}h_{ji})$$

$$a_{ii} = -\frac{1}{W_i}\sum_{j=1}^{n} k_{ij}h_{ij} \qquad (k_{ii}h_{ii} = 0).$$

The solution $\boldsymbol{t}$ depends on the features of the coefficient matrix $\mathbf{A}$.

The properties of $\mathbf{A}$ have been discussed by other authors. For example in [2, 3] it is shown that all the latent roots are real and in [1] it is proved that the necessary and sufficient condition for $\mathbf{A}$ to have at least two zero latent roots is

$$\sum_{i=1}^{n} W_i = 0.$$

It results from the foregoing, that to characterize the spectrum of $\mathbf{A}$ one problem still remains, namely the maximum multiplicity of zero latent roots.

Two lemmas will be helpful in answering this question.

Lemma 1

If $\mathbf{S} = [s_{ij}]$ is a symmetric matrix of order n, $\boldsymbol{x} = (x_1, \ldots, x_n)$ any column vector and $\boldsymbol{y}$ a proper vector of $\mathbf{S}$ corresponding to zero latent root, then the scalar product

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{y}) = 0.$$

The proof depends on the property of a symmetric matrix that

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x}, \mathbf{S}\boldsymbol{y}) = (\boldsymbol{x}, \mathbf{0}) = 0.$$

Lemma 2

If $\boldsymbol{x}$ and $\mathbf{S}$ are given as in lemma 1 and $\mathbf{S}$ is a semidefinite matrix (positive or negative), then only a proper vector, say $\boldsymbol{x}$, of $\mathbf{S}$ appropriate to a zero latent root satisfying

$$\mathbf{S}\boldsymbol{x} = \mathbf{0},$$

may be a non-trivial solution of the equation

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{x}) = (\boldsymbol{x}, \mathbf{S}\boldsymbol{x}) = 0.$$

Proof

Let $\mathbf{S}$ be negative semidefinite. Then it follows that

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{x}) \leqslant 0;$$

furthermore

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{x}) = \sum_{i,j=1}^{n} s_{ij}x_ix_j$$

is a continuous function of the n variables $x_1, x_2, \ldots, x_n$ possessing continuous first and second partial derivatives.